CHSH game with 3 players in a triangle

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1. Introduction

- A Bell inequality [1] is an inequality (or bound) between certain quantities in a given system, whose violation cannot be explained by a classical (local) theory. However, Quantum Mechanics, which is non-local, predicts its violation.
- A very important Bell-type inequality is the Clauser-Horne-Shimony-Holt (**CHSH**) inequality [2]. The CHSH inequality can also be illustrated using a simple game with 2 players and binary inputs and outputs. This game is known as the **CHSH game**.
- In the present poster, the CHSH game is explained and a **proposed extension to 3 players** playing it pairwise in a **triangle** configuration.

2. The CHSH game

The CHSH game has **2 players**: Alice and Bob.

- Binary inputs $x, y \in \{0, 1\}$
- Binary outputs $a, b \in \{0, 1\}$:
- No in-game communication.
- Win if

$$xy = a + b \ (mod\ 2) \tag{1}$$

CHSH GAME

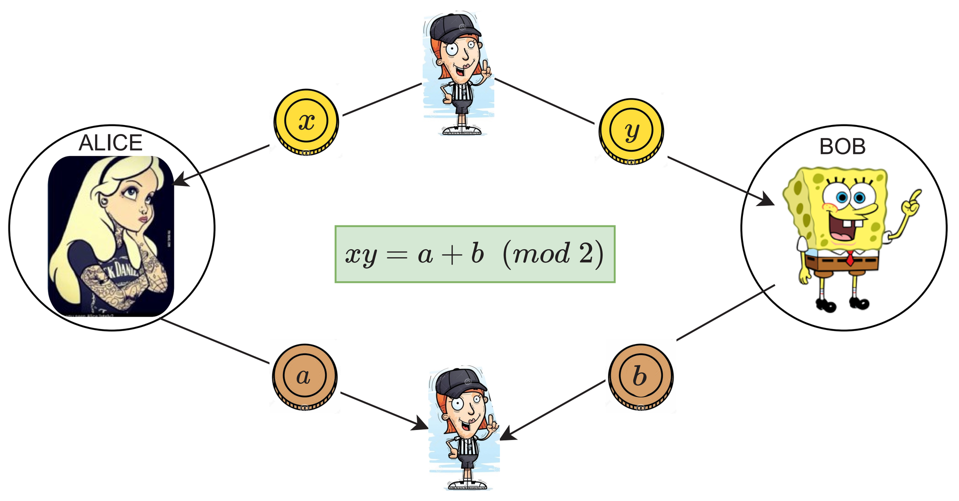


Fig. 1: *Setup of the CHSH game.*

The **classical maximum winning probability** of this game is $3/4 = $ **75%**. If the players use some **quantum** resources (quantum state and measurements; see figure 2), the winning probability can **increase** to **85%**.

Quantum CHSH GAME

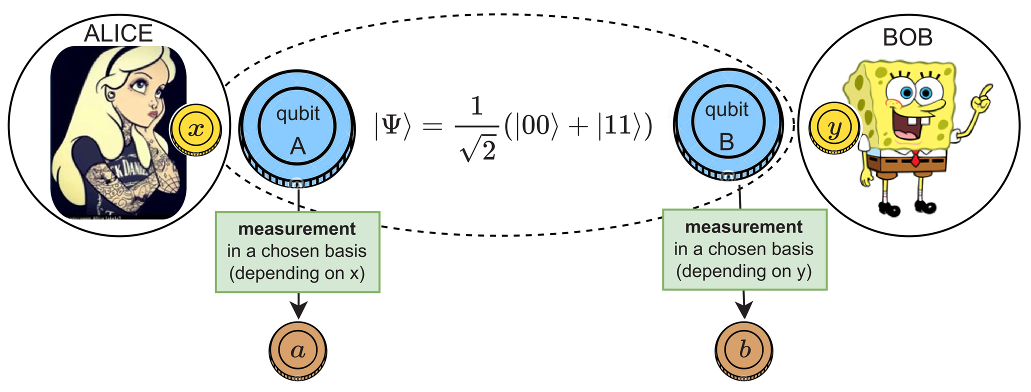
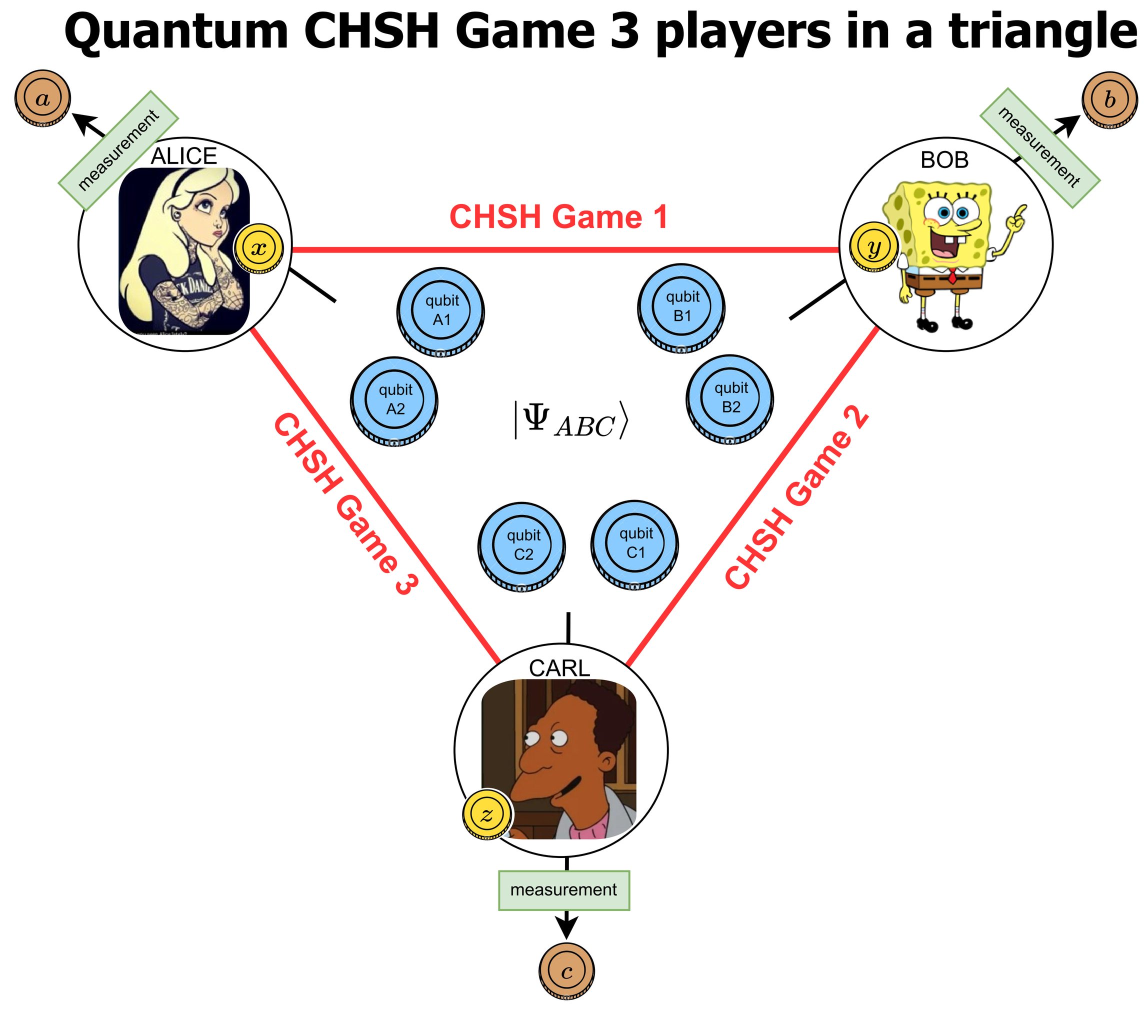


Fig. 2: Players share a maximally entangled state and measure their qubit in a basis that depends on their input. They output the result of the measurement.

3. CHSH game with 3 players in a triangle

Quantum CHSH Game 3 players in a triangle



- 2 CHSH games per player.
- Winning conditions for each game:

$$\text{GAME 1: } xy = a + b \ (mod\ 2) \tag{4}$$

$$\text{GAME 2: } yz = b + c \ (mod\ 2) \tag{5}$$

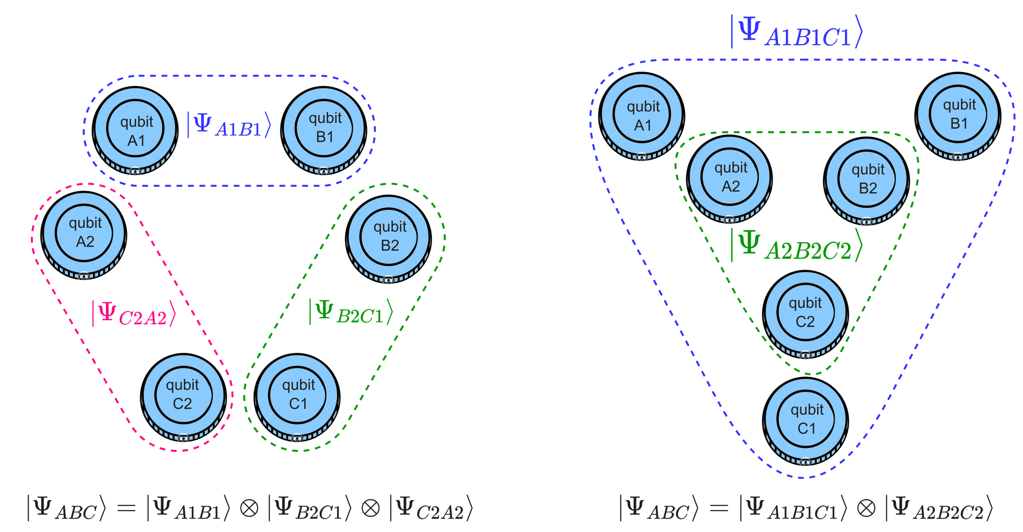
$$\text{GAME 3: } zx = c + a \ (mod\ 2) \tag{6}$$

- Average payoffs from each game.
- **Classically** $\to$ highest win probability is $3/4 =$ **0.75** and lowest $1/4 =$ **0.25**.

QUANTUM

1) **2 qubits per player** $\to$ 6-qubit state $|\Psi_{ABC}\rangle$.

2) Qubits given by three 2-qubit source; *or by* two 3-qubit source. identical sources.



3) Output decided using **projective measurements**. Alice: $\Pi_{x=0}^{a=0} = |a_0\rangle\langle a_0|$ and $\Pi_{x=1}^{a=0} = |\tilde{a}_0\rangle\langle\tilde{a}_0|$, $\Pi_x^{a=1} = \mathbb{I} - \Pi_x^{a=0}$; where:

$$|a_0\rangle = \sqrt{1 - a_{11}^2}\,|00\rangle + a_{11}\,|11\rangle \tag{2}$$

$$|\tilde{a}_0\rangle = \sqrt{1 - \tilde{a}_{11}^2}\,|00\rangle + \tilde{a}_{11}\,|11\rangle \tag{3}$$

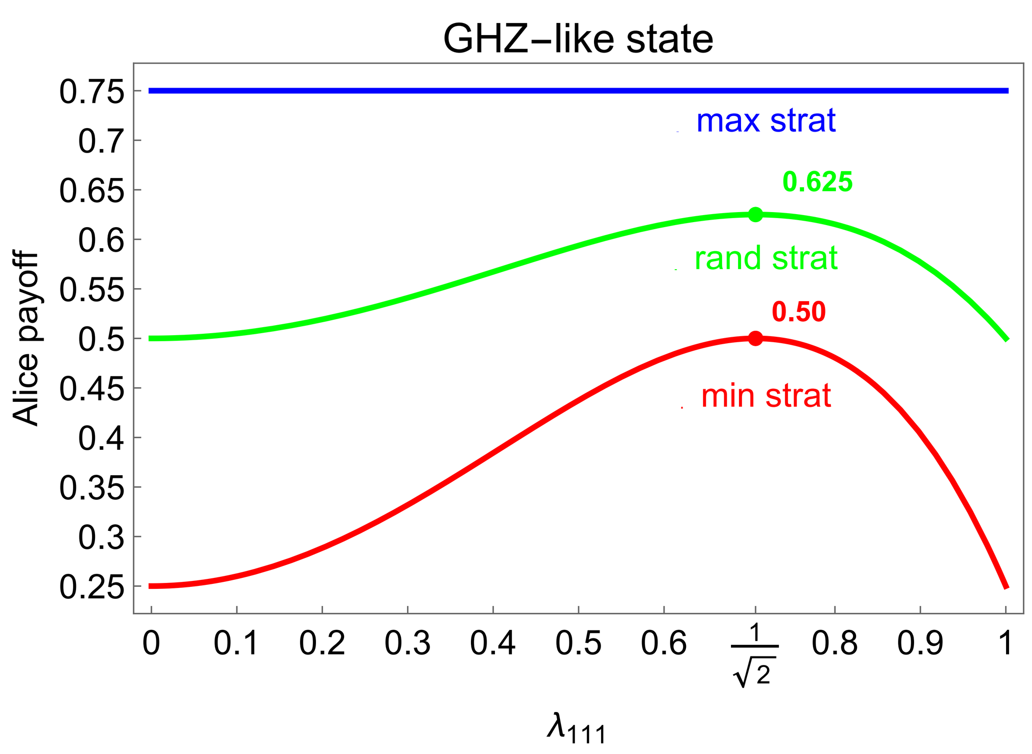
where $a_{11}, \tilde{a}_{11}$ represent Alice's continuous strategy set, $0 \le a_{11}, \tilde{a}_{11} \le 1$.

4) The joint conditional probability of outputs given inputs is:

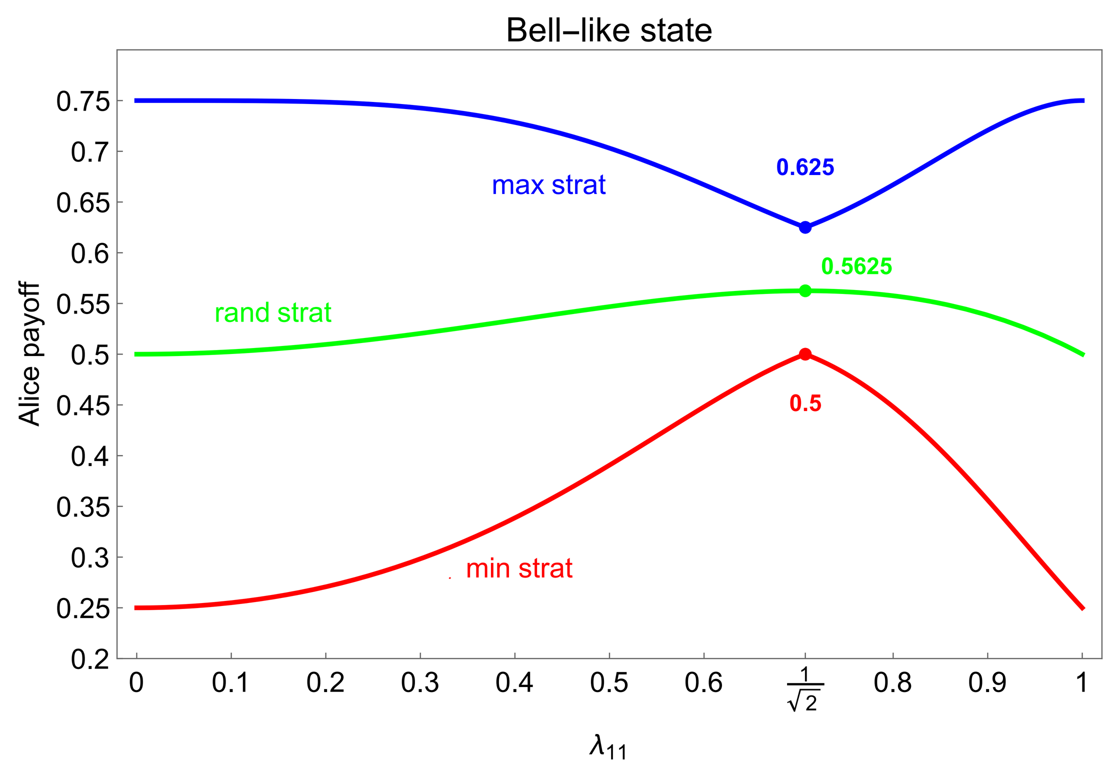
$$P(a, b, c|x, y, z) = \langle\Psi_{ABC}|\,\Pi_x^a \otimes \Pi_y^b \otimes \Pi_z^c\,|\Psi_{ABC}\rangle \tag{7}$$

- Analysed states (all with real coefficients):
  - **GHZ-like** state:
  $$|\Psi_{ABC}\rangle = (\lambda_{000}\,|000\rangle + \lambda_{111}\,|111\rangle)^{\otimes 2} \tag{8}$$
  with $\lambda_{000}^2 + \lambda_{111}^2 = 1$
  - **Bell-like** state:
  $$|\Psi_{ABC}\rangle = (\lambda_{00}\,|00\rangle + \lambda_{11}\,|11\rangle)^{\otimes 3} \tag{9}$$
  with $\lambda_{00}^2 + \lambda_{11}^2 = 1$

4. Results



Alice's payoff is plotted as a function of the entanglement parameter λ_{111} for the **GHZ-like** state in (8). A **max**imising **strat**egy (blue line); a **min**imising strategy (red line); and the average over **random** strategies (green line), which also corresponds to Alice choosing $a_{11} = \tilde{a}_{11} = 1/\sqrt{2}$, regardless of Bob and Carl.



Alice's payoff as a function of λ_{11} for **Bell-like** state in (9). The **max**imising is shown in blue; and the **min**imising in red. The average between the max and min is the green line (**rand**om strategies), which also corresponds to certain strategies.

5. Conclusions

- Overall, the GHZ-like state performs better than the Bell-like state.
- The narrowest difference between max and min is when $\lambda_{11} = \lambda_{111} = 1/\sqrt{2}$.
- The presence of entanglement helps to improve the classical average of 0.5 (see green lines in plot).

References

[1] J. S. Bell. "On the Einstein Podolsky Rosen paradox". In: *Physics Physique Fizika* 1 (3 Nov. 1964).

[2] John F. Clauser et al. "Proposed Experiment to Test Local Hidden-Variable Theories". In: *Phys. Rev. Lett.* 23 (15 Oct. 1969).